Fall 2016 Geometry, Groups, and Dynamics

Nicholas Miller, Purdue University

Title: *Arithmetic progressions in the primitive length spectrum*

Abstract: There have been a host of prime geodesic theorems over the past several decades displaying a surprising analogy between the behavior of primitive, closed geodesics on hyperbolic manifolds and the behavior of the prime numbers in the integers. For instance, just as the prime number theorem dictates the asymptotic growth of the number of primes less than n, there is an analogous asymptotic growth for primitive, closed geodesics of length less than n. In this talk, I will give a brief review of the relevant definitions and go on to give the history of this analogy. I will then discuss some recent work extending this relationship to give the geodesic analogue of the Green–Tao theorem on arithmetic progressions in the prime numbers.

Sagata Mondal, Indiana University

Title: *Geometric and analytic systoles of complete Riemannian surfaces of finite type*

Abstract: The geometric systole, usually called systole, of a complete Riemannian surface S is the minimum of lengths of simple closed geodesics on S that are not homotopic to zero or to a puncture of S. Analytic systole of such a surface S is a quantity obtained by minimizing the ground state energy (first Dirichlet eigenvalue) of certain sub-surfaces of S. In this exposition we shall talk about some relations between the two types of systoles and their occurrence in the study of small eigenvalues of the underlying surface S. This is a joint work with Werner Ballmann, Henrik Matthiesen.

Christopher Leininger, UIUC

Title: *Exotic compact quotients of* $\mathrm{SO}(d,1)$

Abstract: Given a discrete, torsion free subgroup $\Gamma < \mathrm{SO}(d,1)$, the quotient $\Gamma \setminus \mathrm{SO}(d,1)$ is an $\mathrm{SO}(d)$–bundle over the associated hyperbolic d–manifold $\Gamma \setminus \mathbb{H}^d$. For $d = 3$, this is also an example of a complete holomorphic Riemannian manifold of constant curvature. Ghys studied deformations of these structures to a more general class of $\mathrm{SO}(3,1)$ quotients, and this theory was expanded and further studied by Kobayashi and Gueritaud-Kassel. By a result of Tholozan, the volume of compact quotients remains constant under these deformations, and he asked whether there were "exotic" compact quotients, i.e. bundles over compact hyperbolic manifolds with different volume than the standard $\mathrm{SO}(d)$–bundle. Building on a construction of Agol and the work of Gueritaud–Kassel, we describe an infinite family of such exotic compact quotients for $d \leq 4$, and calculate their volumes using Tholozan's volume formula. This is joint work with Grant Lakeland.