Pacific Northwest and Bay Area Differential Geometry Seminar
Saturday and Sunday, February 8-9, 2014
Stanford University
Department of Mathematics Room 380C

The seminar will take place from 10AM to 5PM on Saturday, and 9:15 AM to 1PM on Sunday. Participants and their significant others are invited to a dinner to be arranged at a local restaurant on Saturday evening. The cost of the dinner will be reduced for students and postdocs. Details will be forthcoming on the signup page for the dinner (which you can access by this link: **signup list***).*

List of Speakers

Simon Brendle, Stanford

Otis Chodosh, Stanford

Christine Guenther, Pacific U., Oregon

Mario Micallef, Warwick

Jie Qing, Santa Cruz

Joel Spruck, Johns Hopkins

Directions to the Stanford Mathematics Department are available on the department website. Parking on the campus is plentiful and unrestricted on weekends.

- 10:00–11:00 **Reception, Morning Coffee**

- 11:00-12:00 **Simon Brendle, Stanford**: *A sharp bound for the inscribed radius under mean curvature flow.*

We consider a family of embedded, mean convex hypersurfaces which evolve by the mean curvature flow. It follows from general results of Brian White that the inscribed radius at each point on the hypersurface is at least c/H, where c is a positive constant that depends only on the initial data. Andrews recently gave a new proof of this fact using the maximum principle. In this talk, we will show that the inscribed radius is at least $(1-\delta)/H$ at each point where the curvature is sufficiently large. For $n = 2$, this estimate serves as a substitute for the cylindrical estimates established by Huisken and Sinestrari in the higher dimensional case.

- 12:00–2:00 **Lunch**

There are several places on the Stanford campus that serve lunch. In addition, downtown Palo Alto is a 5-minute drive or a 20-minute walk.
There will be a brief organizational meeting at 1:45.

- 2:00–3:00 **Otis Chodosh, Stanford**: *The renormalized volume and isoperimetric behavior of asymptotically hyperbolic manifolds.*

We will discuss the renormalized volume of asymptotically hyperbolic three-manifolds and a sharp scalar curvature volume comparison result (joint work with Simon Brendle) which bears some

similarities to the Penrose inequality. We will also discuss a recent result showing that large isoperimetric regions in compact perturbations of Schwarzschild-anti-de Sitter are centered coordinate spheres. This has implications for a Bray-style isoperimetric profile approach to the asymptotically hyperbolic Penrose inequality.

• 3:00–4:00 **Afternoon Tea-Coffee**

• 4:00–5:00 **Joel Spruck, Johns Hopkins**: *Interior curvature estimates and the Asymptotic Plateau problem in Hyperbolic space*

We show that for a very general class of curvature functions defined in the positive cone, the problem of finding a complete strictly locally convex hypersurface in H^{n+1} satisfying $f(\kappa) = \sigma \in (0, 1)$ with a prescribed asymptotic boundary Γ at infinity has at least one smooth solution with uniformly bounded hyperbolic principal curvatures. Moreover if Γ is (Euclidean) starshaped, the solution is unique and also (Euclidean) starshaped while if Γ is mean convex the solution is unique. A novel feature of our approach is a "global interior curvature estimate".

• 6:00 **Dinner** (Please sign up using the link **signup list** at the top of the first page.)

Sunday, February 9

• 9:15–9:45 **Morning Coffee**

• 9:45–10:45 **Jie Qing, Santa Cruz**: *Conformal geometry of surfaces in 3-sphere.*

This is a preliminary report for my joint work with Changping Wang and Jingyang Zhong. We are interested in establishing a fundamental theorem for surfaces in conformal 3-sphere and conformal 3-manifolds in general. To do so we regard 3-sphere as the projectivized positive light cone in Minkowski space-time of 5 dimension and, in the same spirit, as the conformal infinity of hyperbolic 4-space. We construct associated surfaces in Minkowski space-time as well as in hyperbolic 4-space and apply fundamental theorem for surfaces in (pseudo)-Riemannian geometry. We are looking to extend the use of ambient spaces of Fefferman and Graham to study the conformal geometry of submanifolds.

• 10:45–11:45 **Christine Guenther, Pacific U., Oregon**: *A geometric introduction to the second order renormalization group flow.*

The Ricci flow arises in physics as the first order approximation of the renormalization group flow for the nonlinear sigma model of quantum field theory. The *second* order approximation is given by the system

$$\frac{\partial}{\partial t} g = -2Rc - \frac{\alpha}{2} Rm^2,$$

and can be considered as a natural nonlinear perturbation of the Ricci flow (here $\alpha > 0$ is a parameter and $Rm^2_{ij} := R_{iklm} R^{klm}_j$). In this talk I will give an introduction to this flow, including recent results from joint work with Jim Isenberg and Karsten Gimre.

• 12:00–1:00 **Mario Micallef, Warwick**: *A gap theorem for the alpha-harmonic maps of Sacks-Uhlenbeck.*

The Mobiüs transformations of the two-sphere are precisely the degree-one minimisers of the Dirichlet energy functional. Among these, the rotations are also minimisers of the alpha-energy. I will describe some joint work with Tobias Lamm and Andrea Malchiodi in which we establish the following gap theorem. Let $E_\alpha(Rot)$ be the alpha-energy of a rotation. There is a fixed $\varepsilon > 0$ and

$\overline{\alpha} > 1$, $\overline{\alpha} - 1$ small, such that, for $1 < \alpha < \overline{\alpha}$, the only degree-one alpha-harmonic maps with alpha- energy less than $E_\alpha(Rot) + \varepsilon$ are the rotations. The most important aspect of this theorem is that ε is *independent of* α. In particular, the dilations of the two-sphere cannot arise as limits of alpha-harmonic maps.